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Dual Ginzburg-Landau Theory for Nonperturbative QCD[★]

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ABSTRACT

Nonperturbative QCD is studied with the dual Ginzburg-Landau theory, where color confinement is realized through the dual Higgs mechanism by QCD-monopole condensation. We obtain a general analytic formula for the string tension. A compact formula is derived for the screened inter-quark potential in the presence of light dynamical quarks. The QCD phase transition at finite temperature is studied using the effective potential formalism. The string tension and the QCD-monopole mass are largely reduced near the critical temperature, T_c . The surface tension is estimated from the effective potential at T_c . We propose also a new scenario of the quark-gluon-plasma creation through the color-electric flux-tube annihilation. Finally, we discuss a close relation between instantons and QCD-monopoles.

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1. Dual Higgs Mechanism for Color Confinement

Color confinement and dynamical chiral-symmetry breaking are quite outstanding features in the nonperturbative QCD.^{1,2} In particular, color confinement is extremely unique, and is characterized by the formation of the color-electric flux tube² with the string tension about 1GeV/fm. To understand the confinement mechanism, much attention has been paid for the analogy between the superconductor and the QCD vacuum.³ Similar to the superconductivity,⁴ the color-electric flux seems to be excluded in the QCD vacuum, which leads the formation of the squeezed color-flux tube between color sources.

In this analogy, color confinement is brought by the dual Meissner effect originated from color-magnetic monopole condensation, which corresponds to Cooper-pair condensation in the superconductivity. As for the appearance of color-magnetic monopoles in QCD, 't Hooft⁵ proposed an interesting idea of the abelian gauge fixing, which is defined by the diagonalization of a gauge-dependent variable. In this gauge, QCD is reduced into an abelian gauge theory with QCD-monopoles, which appear from the hedgehog-like configuration corresponding to the nontrivial homotopy class on the nonabelian manifold, $\pi_2(\text{SU}(N_c)/\text{U}(1)^{N_c-1}) = \mathbb{Z}_{\infty}^{N_c-1}$.

We briefly compare the dual Higgs mechanism in the nonperturbative QCD vacuum with the ordinary Higgs mechanism. Like the Cooper pair in the superconductivity or the Higgs field in the standard theory, the charged-matter field to be condensed is the essential degrees of freedom for the Higgs mechanism. On the other hand, there is only the gauge field in the pure gauge QCD, and hence it seems difficult to find any similarity with the Higgs mechanism. In the abelian gauge, however, only the diagonal gluon remains as the gauge field, and the off-diagonal gluon behaves as the charged-matter field, which leads QCD-monopoles as the relevant degrees of freedom for color confinement. Condensation of QCD-monopoles leads to mass generation of the dual gauge field through the dual Higgs mechanism,^{6,7} which is the dual version of the Higgs mechanism. Thus, the QCD vacuum can be regarded as the dual superconductor after the abelian gauge fixing.

In this framework, the nonperturbative QCD is mainly described by the abelian gauge theory with QCD-monopoles, which is called as the abelian dominance. Many recent studies based on the lattice gauge theory have supported QCD-monopole condensation and the abelian dominance in the maximal abelian gauge.⁸ The dual Higgs scheme predicts the existence of the dual gauge field and the QCD-monopole as the relevant degrees of freedom related to color confinement.^{3,6,7} It can be proved that these particles are color-singlet, so that they can be observed as physical states. The dual gauge field and the QCD-monopole appear as a massive axial-vector glueball and a massive scalar glueball,^{6,7,9} corresponding to the weak vector boson and the Higgs particle in the electro-weak standard theory.

2. Dual Ginzburg-Landau Theory and Inter-Quark Potential

We study the nonperturbative QCD using the dual Ginzburg-Landau (DGL) theory,^{6,7,10} which is an infrared effective theory of QCD based on the dual Higgs mechanism by QCD-monopole condensation.^{6,9} The DGL Lagrangian is described by the diagonal gluon $\vec{A}^\mu \equiv (A_3^\mu, A_8^\mu)$, the dual gauge field $\vec{B}^\mu \equiv (B_3^\mu, B_8^\mu)$ and the QCD-monopole field $\chi_\alpha (\alpha = 1, 2, 3)$,^{6,7}

$$\begin{aligned} \mathcal{L}_{\text{DGL}} = & -\frac{1}{n^2} [n \cdot (\partial \wedge \vec{A})]^\nu [n \cdot^* (\partial \wedge \vec{B})]_\nu - \frac{1}{2n^2} ([n \cdot (\partial \wedge \vec{A})]^2 + [n \cdot (\partial \wedge \vec{B})]^2) \\ & + \bar{q}(i \not{\partial} - e \vec{H} \cdot \vec{A} - m)q + \sum_{\alpha=1}^3 [| (i\partial_\mu - g\vec{\epsilon}_\alpha \cdot \vec{B}_\mu) \chi_\alpha |^2 - \lambda(|\chi_\alpha|^2 - v^2)^2] \end{aligned} \quad (2.1)$$

in the Zwanziger form,¹¹ where the duality of the gauge theory becomes manifest. Here, n_μ corresponds to the direction of the Dirac string, e is the gauge coupling constant, g is the unit magnetic charge obeying the Dirac condition $eg = 4\pi$, and $\vec{\epsilon}_\alpha$ denotes the relative magnetic charge of the QCD-monopole field χ_α .^{6,7} The magnetic charge $g\vec{\epsilon}_\alpha$ is pseudoscalar because of the extended Maxwell equation, $\nabla \cdot \mathbf{H} = \rho_m$. Hence, the dual gauge field \vec{B}_μ is axial-vector. In the absence of matter fields, one finds an exact dual relation between \vec{A}_μ and \vec{B}_μ in the field equation, $\partial \wedge \vec{B} =^* (\partial \wedge \vec{A})$.

In the DGL theory, the self-interaction of the QCD-monopole field χ_α is introduced to realize QCD-monopole condensation. When QCD-monopoles are condensed, the dual Higgs mechanism occurs, and the dual gauge field \vec{B}_μ becomes massive, $m_B = \sqrt{3}gv$. The color-electric field is then excluded in the QCD vacuum through the dual Meissner effect, and is squeezed between color sources to form the hadron flux tube. The QCD-monopole also becomes massive as $m_\chi = 2\sqrt{\lambda}v$.

As for the symmetry of the DGL theory, there is the dual gauge symmetry $[U(1)_3 \times U(1)_8]_m$ corresponding to the local phase invariance of the QCD-monopole field χ_α ^{6,7} as well as the residual gauge symmetry $[U(1)_3 \times U(1)_8]_e$ embedded in $SU(3)_c$. The dual gauge symmetry leads to the conservation of the color-magnetic flux. In the QCD-monopole condensed vacuum, the dual gauge symmetry $[U(1)_3 \times U(1)_8]_m$ is spontaneously broken, and therefore the color-magnetic flux is not conserved. On the other hand, the residual gauge symmetry $[U(1)_3 \times U(1)_8]_e$ is never broken in this process.⁶

We investigate the inter-quark potential in the quenched level using the DGL theory.⁶ By integrating over A_μ and B_μ in the partition functional, the current-current correlation^{6,7} is obtained as $\mathcal{L}_j = -\frac{1}{2}\vec{j}_\mu D^{\mu\nu} \vec{j}_\nu$ with the nonperturbative gluon propagator,

$$D_{\mu\nu} = -\frac{1}{p^2} \left\{ g_{\mu\nu} + (\alpha_e - 1) \frac{p_\mu p_\nu}{p^2} \right\} - \frac{1}{p^2} \frac{m_B^2}{p^2 - m_B^2} \frac{1}{(n \cdot p)^2} \epsilon^\lambda{}_{\mu\alpha\beta} \epsilon_{\lambda\nu\gamma\delta} n^\alpha n^\gamma p^\beta p^\delta \quad (2.2)$$

in the Lorentz gauge. Putting a static quark with color charge^{1,6} \vec{Q} at $\mathbf{x} = \mathbf{r}$ and a static antiquark with color charge $-\vec{Q}$ at $\mathbf{x} = \mathbf{0}$, the quark current is written as $\vec{j}_\mu(x) = \vec{Q} g_{\mu 0} \{\delta^3(\mathbf{x} - \mathbf{r}) - \delta^3(\mathbf{x} - \mathbf{0})\}$. Because of the axial symmetry of the system and the energy minimum condition,⁶, one should take $\mathbf{n} // \mathbf{r}$, which is also used in a similar context of the dual string theory.³ Then, one obtains the inter-quark potential including the Yukawa and the linear parts,^{6,7} $V(r) = -\frac{\vec{Q}^2}{4\pi} \cdot \frac{e^{-m_B r}}{r} + kr$.

To derive the expression for the string tension k , we consider an idealized long flux-tube system, where the field variables can be described by the cylindrical

coordinate $r_T \equiv (x^2 + y^2)^{1/2}$. Like the Abrikosov vortex in the superconductivity,⁴, one should consider the “core” of the hadron flux tubes, where the QCD-monopole condensate $\bar{\chi}$ almost vanishes. The analysis of the field equation shows $\bar{\chi}(r_T) \simeq m_\chi v r_T$ inside the core ($r_T \lesssim m_\chi^{-1}$), and $\bar{\chi}(r_T) \simeq v$ outside the core ($r_T \gtrsim m_\chi^{-1}$). Hence, we adopt the Lorentzian-type ansatz for the QCD-monopole field,

$$\bar{\chi}^2(r_T^2) \simeq v^2 \frac{m_\chi^2 r_T^2}{1 + m_\chi^2 r_T^2} \quad \text{or} \quad \bar{\chi}^2(p_T^2) \simeq v^2 \frac{m_\chi^2}{p_T^2 + m_\chi^2} \quad (2.3)$$

with $p_T \equiv (p_x^2 + p_y^2)^{1/2} \simeq r_T^{-1}$. In this case, we obtain an analytical expression for the string tension,

$$k = \frac{\frac{\bar{Q}^2}{8\pi} m_B^2 m_\chi}{\sqrt{m_\chi^2 - 4m_B^2}} \ln \left(\frac{m_\chi + \sqrt{m_\chi^2 - 4m_B^2}}{m_\chi - \sqrt{m_\chi^2 - 4m_B^2}} \right) = \frac{\frac{\bar{Q}^2}{4\pi} m_B^2 m_\chi}{\sqrt{4m_B^2 - m_\chi^2}} \arccos \frac{m_\chi}{2m_B}. \quad (2.4)$$

For the type-II limit ($m_B \ll m_\chi$), one finds $k \simeq \frac{\bar{Q}^2 m_B^2}{8\pi} \ln(\frac{m_\chi^2}{m_B^2})$,⁶ corresponding to the well-known formula for the energy per unit length of the Abrikosov vortex in the type-II superconductor.⁴

As for the parameter set, the recent lattice QCD studies¹² suggest $m_B \simeq m_\chi$, which means the QCD vacuum corresponds to the dual-superconductor of the type between type I and type II. We show in Fig.1 the inter-quark potential with the choice of $e = 2.0$, $m_\chi = m_B = 1.67\text{GeV}$ corresponding to $\lambda = 29.4$ and $v = 0.154\text{GeV}$, which provide $k \simeq 0.9\text{GeV/fm}$ for the string tension and the radius of the hadron flux as $R \simeq 0.12\text{fm}$. Here, we have included the correction coming from off-diagonal gluons in the short-range part.

3. Infrared Screening Effect by Dynamical Light Quarks

We study the infrared screening effect on the confinement potential due to light quarks.⁶ For instance, a long hadron string can be cut through the light $q\bar{q}$ pair creation, and therefore the inter-quark potential is saturated in the infrared region. Such a tendency is observed in the lattice QCD with dynamical quarks.^{2,13}

We estimate the $q\bar{q}$ pair creation rate w in the color-electric field inside the hadron flux tube, which is formed between valence quarks. Using the Schwinger formula^{6,14} in QCD, we estimate the expectation value of the energy of the created $q\bar{q}$ pair as $\langle E_{q\bar{q}} \rangle \simeq 0.85\text{GeV}$. Since the energy $\langle E_{q\bar{q}} \rangle$ is supplied by the missing length of the hadronic string, the infrared screening length R_{sc} can be estimated from $kR_{\text{sc}} \sim \langle E_{q\bar{q}} \rangle$. Thus, one obtains $R_{\text{sc}} \sim 1\text{fm}$, which corresponds to a typical value of the hadron size.⁶

The hadronic string becomes unstable against the $q\bar{q}$ pair creation when the distance between the valence quarks becomes larger than R_{sc} . This means the vanishing of the strong correlation between the valence quarks in the infrared region, so that the corresponding infrared cutoff, $a \simeq R_{\text{sc}}^{-1} \sim 200\text{MeV}$, should appear in the system.⁶ Taking account of the infrared screening effect, we introduce the infrared cutoff a to the nonperturbative gluon propagator (2.2) by replacing $\frac{1}{(n \cdot p)^2} \rightarrow \frac{1}{(n \cdot p)^2 + a^2}$,⁶ because the non-local factor $\langle x | \frac{1}{(n \cdot p)^2} | y \rangle$ provides the strong and long-range correlation as the origin of the confinement potential.⁶ Here, this gluon propagator keeps the residual gauge symmetry. Such a disappearance of the infrared double pole in the gluon propagator in the DGL theory can be qualitatively shown by considering the polarization diagram of quarks.

Using the above gluon propagator, we obtain a compact formula⁶ for the quark potential including the infrared screening effect by the $q\bar{q}$ pair creation,

$$V_{\text{sc}}(r) = -\frac{\vec{Q}^2}{4\pi} \cdot \frac{e^{-m_B r}}{r} + k \cdot \frac{1 - e^{-ar}}{a}, \quad (3.1)$$

which exhibits the saturation for the longer distance than $a^{-1} \simeq 1\text{fm}$. This formula for the screened quark potential has been used not only for the lattice QCD results

with light dynamical quarks², but also for the phenomenological analysis of the hadron decay.¹⁵

4. QCD Phase Transition at Finite Temperature

We study the QCD vacuum at finite temperature¹⁶ using the DGL theory^{17,18} at the quenched level, where the quark degrees of freedom are frozen. In this case, the quark term can be dropped in the DGL Lagrangian, and therefore one obtains a simple partition functional^{17,18} after the integration over the gauge field A_μ ,

$$Z[J] = \int \mathcal{D}\chi_\alpha \mathcal{D}\vec{B}_\mu \exp \left(i \int d^4x \{ \mathcal{L}_{\text{DGL}}^{\text{quench}} - J \sum_{\alpha=1}^3 |\chi_\alpha|^2 \} \right),$$

$$\mathcal{L}_{\text{DGL}}^{\text{quench}} \equiv -\frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \sum_{\alpha=1}^3 [|(i\partial_\mu - g\vec{\epsilon}_\alpha \cdot \vec{B}_\mu)\chi_\alpha|^2 - \lambda(|\chi_\alpha|^2 - v^2)^2].$$
(4.1)

Here, we have introduced the quadratic source term.^{17,18} The thermodynamical potential is then obtained as

$$V_{\text{eff}}(\bar{\chi}; T) = 3\lambda(\bar{\chi}^2 - v^2)^2 + 3\frac{T}{\pi^2} \int_0^\infty dk k^2 \ln \left(1 - e^{-\sqrt{k^2 + m_B^2}/T} \right)$$

$$+ \frac{3}{2} \frac{T}{\pi^2} \int_0^\infty dk k^2 \ln \left(1 - e^{-\sqrt{k^2 + m_\chi^2}/T} \right).$$
(4.2)

The glueball masses m_χ and m_B depend on the QCD-monopole condensate $\bar{\chi}$,

$$m_\chi^2(\bar{\chi}) = 2\lambda(3\bar{\chi}^2 - v^2) + J(\bar{\chi}) = 4\lambda\bar{\chi}^2, \quad m_B^2(\bar{\chi}) = 3g^2\bar{\chi}^2, \quad (4.3)$$

which are always non-negative for the whole region of $\bar{\chi}$. Owing to the introduction of the quadratic source term in Eq.(4.1), we can formulate the effective action for the whole region of the order parameter without any difficulty of the imaginary scalar-mass problem^{16,17,18} in the ϕ^4 -type theory.

Like the Ginzburg-Landau theory in the superconductivity,⁴ one should consider the possibility of the T -dependence on the parameters (λ, v) in the DGL theory. In particular, the self-interaction of χ_α is introduced phenomenologically, and it should be reduced at high T according to the asymptotic freedom behavior of QCD. Hence, we adopt a simple ansatz for the T -dependence on λ ,^{17,18} $\lambda(T) \equiv \lambda(1 - \alpha T/T_c)$. (We take $\lambda(T) = 0$ for $T > T_c/\alpha$.) We take $\alpha \simeq 0.97$ to reproduce the thermodynamical critical temperature $T_c \simeq 0.2\text{GeV}$, which means a large reduction of the self-interaction among QCD monopoles near T_c . Our results to be shown below do not depend largely on the value of T_c , which may takes a larger value, e.g. $T_c \simeq 0.26\text{GeV}$, suggested from the recent lattice QCD simulations¹⁹.

We find a first-order phase transition at $T_c=0.2\text{GeV}$. The lower and upper critical temperatures are $T_{\text{low}} = 0.163\text{GeV}$ and $T_{\text{up}} = 0.205\text{GeV}$, respectively. We show in Fig.2 the glueball masses $m_B(T)$ and $m_\chi(T)$ at finite temperature. A large reduction of the glueball mass is suggested near T_c . In particular, the QCD-monopole mass $m_\chi(T)$ largely drop down to $m_\chi \sim T_c (\simeq 0.2\text{GeV})$ near T_c . The deconfinement phase transition occurs at the temperature satisfying $m_\chi \simeq T$, which seems quite natural because only low-lying modes with $\omega_n \lesssim T$ can contribute significantly due to the thermodynamical factor $1/(e^{\omega_n/T} \pm 1)$. Similar glueball-mass reduction is also suggested by the thermodynamical studies based on the lattice QCD data.²⁰

We show in Fig.3 the string tension $k(T)$ at finite temperature, calculated by using Eq.(2.4). The string tension $k(T)$ decreases rapidly with temperature, and drops down to zero around $T_c = 0.2 \text{ GeV}$. Hence, one expects a rapid change of the masses and the sizes of the quarkonia according to the large reduction of $k(T)$ near T_c . Our result agrees with the lattice QCD data in the pure gauge²¹: $k(T) \simeq k(0)(1 - T/T_c)^{0.42}$.

We estimate the surface tension σ between the confinement and deconfinement phases using the effective potential at T_c in the DGL theory. There are two minima

at $\bar{\chi} = 0$ and $\bar{\chi} = \bar{\chi}_c$ in $V_{\text{eff}}(\bar{\chi}; T_c)$. The mixed phase includes both the confinement phase ($\bar{\chi} = \bar{\chi}_c$) and the deconfinement phase ($\bar{\chi} = 0$). When the boundary surface in the mixed phase is taken on the xy -plane ($z = 0$), the system depends only on the z -coordinate, and the boundary condition is given as $\bar{\chi}(z = -\infty) = 0$, $\bar{\chi}(z = \infty) = \bar{\chi}_c$. The surface tension σ in the DGL theory is estimated as

$$\sigma \simeq \int_{-\infty}^{\infty} dz \left\{ 3 \left(\frac{d\bar{\chi}(z)}{dz} \right)^2 + V_{\text{eff}}[\bar{\chi}(z); T_c] \right\}. \quad (4.4)$$

We approximate the figure of $V_{\text{eff}}(\bar{\chi}; T_c)$ ($0 \leq \bar{\chi} \leq \bar{\chi}_c$) as a sine curve, $V_{\text{eff}}(\bar{\chi}; T_c) \simeq \frac{h}{2} \{1 - \cos(2\pi\bar{\chi}/\bar{\chi}_c)\}$, where h and $\bar{\chi}_c$ corresponds to “height” and “width” of the “potential barrier” between the two stable states at T_c . The field equation of $\bar{\chi}(z)$ is then solved analytically like the sine-Gordon equation,²²

$$\bar{\chi}(z) \simeq \frac{2\sqrt{6}}{3} \tan^{-1} e^{z/\delta}, \quad \delta \equiv \frac{\sqrt{3}}{\pi} \bar{\chi}_c / \sqrt{h}, \quad \sigma \simeq \frac{4\sqrt{3}}{\pi} \sqrt{h} \bar{\chi}_c, \quad (4.5)$$

where δ denotes the thickness of the boundary between the two phases. In terms of the effective potential, the smallness of σ corresponds to the weakness of the first-order phase transition, because σ takes smaller value for smaller “height” h or “width” $\bar{\chi}_c$ of the “potential barrier” in $V_{\text{eff}}(\bar{\chi}; T_c)$.

One finds $\bar{\chi}_c \simeq 0.75\text{fm}^{-1}$ and $h \simeq 0.33\text{fm}^{-4}$ from $V_{\text{eff}}(\bar{\chi}; T_c)$. Hence, the surface tension is estimated as $\sigma^{1/3} \simeq 196\text{MeV}$, and the thickness of the border between the two phases is $\delta \simeq 0.7\text{fm}$. Since the above estimation has been done in the quenched level, the obtained results are to be compared with the lattice QCD data in the quenched level, e.g. $\sigma^{1/3} \sim 80\text{MeV}$.²³ Therefore, our estimation for $\sigma^{1/3}$ seems rather good in spite of the rough treatment.

5. Application to Quark-Gluon-Plasma Physics

We apply the DGL theory to the quark-gluon-plasma (QGP) physics in ultrarelativistic heavy-ion collisions. In a standard picture of the QGP formation, many color-electric flux tubes are formed between heavy ions immediately after the collision.^{14,24} In this pre-equilibrium stage, there occurs $q\bar{q}$ pair creation violently inside tubes.^{14,24}, and the energy of the color-electric field turns into that of the stochastic kinetic motion of quarks (and gluons). The energy deposition and the thermalization thus occur.

We here examine the interaction between the color-electric flux tubes in the DGL theory to study the QGP formation in terms of the flux-tube dynamics, because many flux tubes would overlap in the central region between heavy ions just after collisions. There are several kinds of flux tubes in the QCD system. Each flux tube is characterized by the color charge^{1,6} \vec{Q} at its end.

We study the interaction between two color-electric flux tubes with the color-electric charge \vec{Q}_1 and \vec{Q}_2 at their ends. The system is idealized as two sufficiently long flux tubes, where the separation distance between them is denoted by d . For $d \gg m_\chi^{-1}$, the interaction energy per unit length in this system is estimated as¹⁷ $E_{\text{int}} \simeq \frac{\vec{Q}_1 \cdot \vec{Q}_2}{2\pi} m_B^2 K_0(m_B d)$ using the similar calculation for the Abrikosov vortex in the superconductor.⁴

There are two interesting cases on the interaction between two color-electric flux tubes.

- (a) For the same flux tubes with opposite flux direction (e.g. $R\bar{R}$ and $\bar{R}R$), one finds $\vec{Q}_1 = -\vec{Q}_2$ i.e. $\vec{Q}_1 \cdot \vec{Q}_2 = -e^2/3$, so that these flux tubes are attracted each other, and would be annihilated into dynamical gluons.
- (b) For the different flux tubes satisfying $\vec{Q}_1 \cdot \vec{Q}_2 < 0$ (e.g. $R\bar{R}$ and $B\bar{B}$), one finds $\vec{Q}_1 \cdot \vec{Q}_2 = -e^2/6$, so that these flux tubes are attractive, and would be unified into a single flux tube (similar to $\bar{G}G$ flux tube).

Based on the above calculation, we propose a new scenario of the QGP formation via the annihilation of the color-electric flux tubes.¹⁷ When the flux tubes are sufficiently dense in the central region after the collisions, many flux tubes are annihilated or unified. During their annihilation process, lots of dynamical gluons (and quarks) would be created, and the energy of the flux tubes turns into that of the random kinetic motion of gluons (and quarks). The thermalization is achieved through the stochastic gluon collisions, and finally a hot QGP would be created. Here, the gluon self-interaction in QCD plays an essential role to the thermalization process, which is quite different from the photon system in QED.

In more realistic case, both the quark-pair creation and the flux-tube annihilation would take place at the same time. For instance, the flux tube breaking^{14,24} would occur before the flux tube annihilation for the dilute flux tube system. On the other hand, in case of extremely high energy collisions, there would be lots of flux tubes overlapping in the central region between heavy ions, and therefore the flux tube annihilation should play the dominant role in the QGP formation. In any case, the DGL theory would provide a calculable method for the dynamics of the color-electric flux tubes in the QGP formation.

6. Relation between Instanton and QCD-monopole Trajectory

Finally, we study the relation between the QCD-monopole and the instanton²², which is another important topological object in nonabelian gauge theories. There is an ambiguity on the gauge-dependent variable $X(x)$ to be diagonalized in the abelian gauge fixing, and therefore we choose a suitable $X(x)$ to describe the instanton configuration. The Polyakov gauge, where $A_4(x)$ is to be diagonalized, is very interesting, because $A_4(x)$ takes the hedgehog-like configuration near the well-localized instanton, and the QCD-monopole trajectory should pass through its center inevitably. Here, we show this relation in the Euclidean $SU(2)$ -gauge theory.

The gauge configuration near the well-localized instanton is given by

$$A_\mu(x) \simeq -i\eta^{a\mu\nu}\sigma^a \frac{(x-x_0)^\nu}{|(x-x_0)|^2 + a_0^2} \quad (6.1)$$

in the non-singular gauge.²² Here, $\eta^{a\mu\nu}$ is the 't Hooft symbol; $x_0^\mu \equiv (\mathbf{x}_0, t_0)$ and a_0 denote the center coordinate and the size of the instanton, respectively. In particular, one finds

$$A_4(x) \simeq i \frac{\sigma^a(\mathbf{x} - \mathbf{x}_0)^a}{|(x-x_0)|^2 + a_0^2}, \quad (6.2)$$

so that there inevitably appears a QCD-monopole trajectory with temporal direction penetrating the center of the instanton in the Polyakov gauge. For instance, the QCD-monopole trajectory $x_\mu \equiv (\mathbf{x}, t)$ is exactly found as

$$\mathbf{x} = \mathbf{x}_0, \quad -\infty < t < \infty \quad (6.3)$$

for the one-instanton solution at the classical level. Thus, the QCD-monopole trajectory is expected to have a close relation to the instanton configuration.²⁵

We find an interesting feature of the QCD-monopole trajectory in the Polyakov gauge in the multi-instanton solution²²,

$$A_\mu(x) = -i\bar{\eta}^{a\mu\nu}\sigma^a \left(\sum_j \frac{a_j^2(x-x_j)^\nu}{|x-x_j|^4} \right) / \left(1 + \sum_k \frac{a_k^2}{|x-x_k|^2} \right). \quad (6.4)$$

For instance, there appear two junctions and a loop in the QCD-monopole trajectory in the two-instanton system as shown in Fig.4. Here, we consider the two instantons with the same size locating at $(\pm x_0, 0, 0, 0)$ for simplicity. In this case, the QCD-monopole trajectory is found to be $(x, 0, 0, t)$ with

$$x = 0 \quad \text{or} \quad t^2 = (x_0^2 - x^2) + 2|x_0|\sqrt{(x_0^2 - x^2)}. \quad (6.5)$$

The QCD-monopole trajectories tend to be highly folded by connecting their loops in the multi-instanton configuration. Hence, the presence of instantons is expected

to promote QCD-monopole condensation, which is characterized by a folded long monopole-loop⁸. This conjecture can be checked by the lattice QCD.

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FIGURE CAPTIONS

- 1) The inter-quark potential $V(r)$ in the dual Ginzburg-Landau theory. The dashed curve denotes the Cornell potential.
- 2) The glueball masses $m_B(T)$ and $m_\chi(T)$ at finite temperature. A large glueball-mass reduction is found near T_c . The phase transition occurs at the temperature satisfying $m_\chi \simeq T$, which is denoted by the dotted line.
- 3) The string tension $k(T)$ at finite temperature T . The lattice QCD result in the pure gauge in Ref.[21] is shown by the dashed curve.
- 4) in the two-instanton system in the Polyakov gauge. The two instantons with the same size are located at $(\pm x_0, 0, 0, 0)$ shown by small circles. There appear two junctions and a loop in the QCD-monopole trajectory.